Counting fuzzy subgroups of S2 S3

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COUNTING FUZZY SUBGROUPS OF SYMMETRIC GROUPS S_2, S_3 AND ALTERNATING GROUP A_4

(Membilang Subkumpulan Kabur bagi Kumpulan Simetrik S_2, S_3
dan Kumpulan Selang Seli A_4)

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ABSTRACT

In this article we compute the number of fuzzy subgroups of symmetric groups S_2, S_3 and alternating group A_4. First, we define an equivalence relation on the set of all fuzzy subgroups of a group G. We represent all of the subgroups of S_2, S_3 and A_4. We use their poset diagrams to determine the number of their fuzzy subgroups.

Keywords: equivalence; fuzzy subgroup; permutation group

ABSTRAK

Dalam makalah ini akan dibilang subkumpulan kabur daripada kumpulan-kumpulan simetrik S_2, S_3 dan kumpulan selang seli A_4. Pertamanya ditakrifkan hubungan kesetaraan pada set semua subkumpulan kabur daripada suatu kumpulan G. Semua subkumpulan daripada S_2, S_3 dan A_4 akan diwakilkan. Gambar rajah poset digunakan untuk menentukan bilangan subkumpulan kaburnya.

Kata kunci: kesetaraan; subkumpulan kabur; kumpulan permutasi

1. Introduction

After the first paper on fuzzy groups in 1971 by Rosenfeld, researchers started to work on characterizing fuzzy subgroups of various groups. Without any equivalence relation on fuzzy subgroups the number of fuzzy subgroups is infinite even for the trivial group. So we define and study an equivalence relation on the set of all fuzzy subgroups of a given set. Using this equivalence classes of fuzzy subgroups, we determine the number of distinct equivalence classes of fuzzy subgroups of symmetric groups S_2, S_3 and alternating group A_4. This definition is different from Murali and Makamba (2001), but equivalent to the definition of Dixit et al. (1996).

Murali and Makamba (2001, 2003, 2004), Zhang and Zou (1998) computed the number of fuzzy subgroups of some abelian groups and cyclic groups. This paper gives a guiding principle to compute the number of fuzzy subgroups of nonabelian groups S_2, S_3 and alternating group A_4.

2. Preliminaries

We recall some definitions and results which will be used later.

Definition 2.1. A *partial order* on a nonempty set P is a binary relation $\leq$ on P with reflexive, antisymmetric and transitive properties. The pair $\langle P, \leq \rangle$ is called a *partially ordered set* or *poset.* A nonempty subset S of P is a chain in P if S is totally ordered by $\leq$. A finite chain with n elements can be written in the form $c_1 < c_2 < . . . < c_n$ (Roman 2008).

Definition 2.2. (Zadeh 1965). Let X be a nonempty set. A *fuzzy subset* of X is a function μ from X into $[0,1]$.

Definition 2.3. (Rosenfeld 1971). Let G be a group. A fuzzy subset μ of G is called a *fuzzy subgroup* of G if

(1) $\mu(xy) \geq \min\{\mu(x), \mu(y)\}, \forall x, y \in G$,
(2) $\mu(x^{-1}) \geq \mu(x), \forall x \in G$.

Theorem 2.1. (Antony & Sherwood 1979). *Let e denote identity element of group G. If μ is a fuzzy subgroup of G, then*

(1) $\mu(e) \geq \mu(x), \forall x \in G$,
(2) $\mu(x^{-1}) = \mu(x), \forall x \in G$.

In this paper we always denote θ_i as element of $[0,1]$ and $\theta_i > \theta_j$ if $i < j$.
If μ is a fuzzy subgroup of finite group G, then $Im(\mu) = \{\mu(x) | x \in G\}$ is finite. By using Theorem 2.1, μ must be in the form

$$\mu(x) = \begin{cases} \theta_1 & x \in [\{e\} \cup A] = B_1, \exists A \subset G \\ \theta_2 & x \in B_2 \\ & \vdots \\ \theta_p & x \in B_p \end{cases}, \qquad (1)$$

with $\bigcup_{i=1}^{p} B_i = G$ and $B_i \cap B_j = \emptyset$ for $i \neq j$.

Let $P_m = \bigcup_{i=1}^{m} B_i, m \leq p$. Using this symbol we have,
$B_i = \{x \in G | \mu(x) = \theta_i\}$, $P_j = \{x \in G | \mu(x) \geq \theta_j\}$, $B_{i+1} = P_{i+1} \setminus P_i$ and $P_p = \bigcup_{i=1}^{p} B_i = G$.

Theorem 2.2. *Function μ as in* (1) *is a fuzzy subgroup of G if only if P_m is a subgroup of G,* $\forall m \in \{1,2,\ldots,p\}$.

Proof: Let μ be a fuzzy subgroup of G and m be any arbitrary element in $\{1,2,\ldots, p\}$.
Note that $0 \in P_1$ and hence $0 \in P_m$.
If $x, y \in P_m$, then $\mu(x) \geq \theta_m$ and $\mu(y) \geq \theta_m$. Thus

$\mu(xy) \geq \min\{\mu(x), \mu(y)\}$
$\mu(xy) \geq \theta_m$, that is $xy \in P_m$.

From Theorem 2.1.(2), $\mu(x^{-1}) = \mu(x) \geq \theta_m$. Hence $x^{-1} \in P_m$.
Thus, P_m is a subgroup of G.
Now, let P_m be a subgroup of G, $\forall m \in \{1,2,\ldots, p\}$ and let $x, y \in G$ and $\mu(x) = \theta_k$, $\mu(y) = \theta_m$ for some natural numbers k and m with $k < m$.
We have $x \in P_k$, $y \in P_m$. Since $k < m$, we have $x, y \in P_m$. Since P_m is a subgroup of G, then $xy \in P_m$.
Therefore, $\mu(xy) \geq \theta_m = \mu(y)$. Also $\theta_m < \theta_k = \mu(x)$.
Thus,

$$\mu(xy) \geq \min\{\mu(x), \mu(y)\}. \qquad (2)$$

On the other hand, if $x \in G$ and $x \in B_i$ for some $i \in \{1,2,\ldots, p\}$, then $\mu(x) = \theta_i$. Therefore $x \in P_i$. Since P_i is a subgroup of G, then $x^{-1} \in P_i$. Thus,

$$\mu(x^{-1}) \geq \theta_i = \mu(x). \qquad (3)$$

Using (2) and (3), we conclude that μ is a fuzzy subgroup of G. □

Corollary 2.1. *Function $\mu: G \to [0,1]$ is a fuzzy subgroup of G if there is a chain $P_1 \subset P_2 \subset \dots \subset P_n = G$ in poset $S(G)$ such that*

$$\mu(x) = \begin{cases} \theta_1 & x \in P_1 \\ \theta_2 & x \in P_2 \setminus P_1 \\ & \vdots \\ \theta_n & x \in P_n \setminus P_{n-1} \end{cases}. \tag{4}$$

We define the *length* (or *order*) of the fuzzy subgroup μ in (4) to be n and we write $|\mu|$.

3. Equivalence Relation on Fuzzy Subgroups

Without any equivalence relations on fuzzy subgroups of a group G, the number of fuzzy subgroups is infinite even for the trivial group. So we define equivalence relation on the set of all fuzzy subgroups of a given group. Using this equivalent, we compute the number of distinct equivalence classes of fuzzy subgroups of symmetric groups S_2, S_3 and alternating group A_4 .

Definition 3.1. Let μ, γ be fuzzy subgroups of G of the form

$$\mu(x) = \begin{cases} \theta_1 & x \in P_1 \\ \theta_2 & x \in P_2 \setminus P_1 \\ & \vdots \\ \theta_n & x \in P_n \setminus P_{n-1} \end{cases}, \gamma(x) = \begin{cases} \delta_1 & x \in M_1 \\ \delta_2 & x \in M_2 \setminus M_1 \\ & \vdots \\ \delta_m & x \in M_m \setminus M_{m-1} \end{cases}.$$

Then we say that μ and γ are *equivalent* and write $\mu \sim \gamma$, if

(1) $m = n$

(2) $P_i(\mu) = M_i(\gamma), \forall i \in \{1,2,\dots, n\}$.

It is easily checked that this relation is indeed an equivalence relation. Two fuzzy subgroups of G are said to be different if they are not equivalent.

Example 3.1. Consider the group $G = Z_{12}$. Let $\mu, \gamma, \alpha, \beta$ be functions from Z_{12} into $[0,1]$ such that

$$\mu(x) = \begin{cases} 1 & x \in \{0,2,4,6,8,10\} \\ 1/2 & x \in \{1,3,5,7,9,11\} \end{cases}, \gamma(x) = \begin{cases} 1 & x \in \{0,1,2,3,4,5\} \\ 1/2 & x \in \{6,7,8,9,10,11\} \end{cases},$$

$$\alpha(x) = \begin{cases} 1 & x \in \{0,4,8\} \\ 1/2 & x \in \{2,6,10\} \\ 1/3 & x \in \{1,3,5,7,9,11\} \end{cases}, \beta(x) = \begin{cases} 1/2 & x \in \{0,4,8\} \\ 1/4 & x \in \{2,6,10\} \\ 0 & x \in \{1,3,5,7,9,11\} \end{cases}.$$

Note that $P_1(\mu) = \{0,2,4,6,8,10\}$, $P_2(\mu) = \{0,2,4,6,8,10\} \cup \{1,3,5,7,9,11\} = Z_{12}$. Thus $P_1(\mu), P_2(\mu)$ are both subgroups of Z_{12}. By Theorem 2.2, μ is a fuzzy subgroup of Z_{12}. Similarly, we can show that α and β both are fuzzy subgroups of Z_{12}.

Note that $P_1(\gamma) = \{0,1,2,3,4,5\}$ is not a subgroup of Z_{12}, so γ is not a fuzzy subgroup of Z_{12}.
Since $|\mu| \neq |\alpha|$ and $|\mu| \neq |\beta|$, by Definition 3.1 we have $\mu \nsim \alpha$ and $\mu \nsim \beta$. Note that $|\alpha| = |\beta|$ and it is easy to show that $P_i(\alpha) = P_i(\beta), \forall i \in \{1,2,3\}$. Thus $\alpha \sim \beta$.

4. The Number of Fuzzy Subgroups of Symmetric Groups S_2, S_3 and Alternating Group A_4

In this section, we give a guiding principle to determine the number of fuzzy subgroups of S_2, S_3 and alternating group A_4. We denote the number of fuzzy subgroups of group G by $o(F_G)$.

4.1. *The Number of Fuzzy Subgroups of* S_2

Let $S_2 = \{\varphi_1 = id = e, \varphi_2 = (12)\}$. The two subgroups of S_2 are $\{\varphi_1\}$ and S_2. Therefore, we can construct a fuzzy subgroup μ of S_2 with length of μ equal to 1 or 2. The fuzzy subgroup of S_2 with length 1 is $\mu(x) = \theta_1, \forall x \in S_2$.
While the fuzzy subgroup of S_2 with length 2 is

$$\mu(x) = \begin{cases} \theta_1 & x \in \{e\} \\ \theta_2 & x \in S_2 \backslash \{e\} \end{cases}.$$

Thus, $o(F_{S_2}) = 2$.

4.2. *The Number of Fuzzy Subgroups of* S_3

Let $S_3 = \{\varphi_1 = id = e, \varphi_2 = (23), \varphi_3 = (12), \varphi_4 = (123), \varphi_5 = (13), \varphi_6 = (132)\}$. There are six subgroups of S_3, namely S_3, $H_1 = \{\varphi_1\}$, $H_2 = \{\varphi_1, \varphi_2\}$, $H_3 = \{\varphi_1, \varphi_3\}$, $H_4 = \{\varphi_1, \varphi_5\}$, and $H_5 = \{\varphi_1, \varphi_4, \varphi_6\}$. Let $S(G)$ denotes the set of all subgroups of group G. The diagram of poset $\langle S(S_3), \subset \rangle$ is as follows:

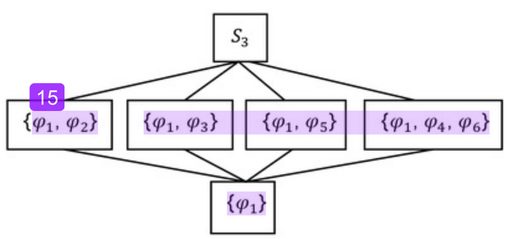

Figure 1: Diagram of poset subgroups of S_3

We will compute the number of fuzzy subgroups of S_3. Let μ be a fuzzy subgroup of S_3. We will identify μ according to $P_1(\mu)$. Every subgroup of S_3 can be chosen to be $P_1(\mu)$. If $P_1(\mu) = S_3$, we only have one fuzzy subgroup of S_3 that is $\mu(x) = \theta_1, \forall x \in S_3$.
If $P_1(\mu) = H_2$, then the only option we have is $S_3 = P_2(\mu)$. Therefore we only have one fuzzy subgroup of S_3 namely

$$\mu(x) = \begin{cases} \theta_1 & x \in H_2 \\ \theta_2 & x \in S_3 \backslash H_2 \end{cases}.$$

Similarly, we have one fuzzy subgroup for $P_1(\mu) = H_3, P_1(\mu) = H_4, P_1(\mu) = H_5$, namely

$$\mu(x) = \begin{cases} \theta_1 & x \in H_3 \\ \theta_2 & x \in S_2 \setminus H_3 \end{cases}, \mu(x) = \begin{cases} \theta_1 & x \in H_4 \\ \theta_2 & x \in S_3 \setminus H_4 \end{cases}, \mu(x) = \begin{cases} \theta_1 & x \in H_5 \\ \theta_2 & x \in S_3 \setminus H_5 \end{cases},$$

respectively.

Finally if $P_1(\mu) = H_1$, then by observing the poset of S_3 (see figure 1) we may construct fuzzy subgroups of S_3 of length 2 or 3. If the length is 2, then $P_2(\mu) = S_3$. If the length is 3, then $P_3(\mu) = S_3$ and we can choose one out of four to be $P_2(\mu)$, namely H_2, H_3, H_4 and H_5. Thus, there are five fuzzy subgroups that can be constructed with $P_1(\mu) = H_1$. Thus the total number of fuzzy subgroups of S_3 is ten.

4.3. *The Number of Fuzzy Subgroups of Alternating Group A_4*

Let $S_4 = \{e = id, \sigma_1, \sigma_2, \ldots, \sigma_9, \tau_1, \tau_2, \ldots, \tau_8, \alpha_1, \alpha_2, \ldots, \alpha_6\}$ with

$$\sigma_1 = (1234), \sigma_2 = (13)(24), \sigma_3 = (1432),$$
$$\sigma_4 = (1243), \sigma_5 = (14)(23), \sigma_6 = (1342), \sigma_7 = (1324),$$
$$\sigma_8 = (12)(34), \sigma_9 = (1423), \tau_1 = (234), \tau_2 = (243),$$
$$\tau_3 = (134), \tau_4 = (143), \tau_5 = (124), \tau_6 = (142),$$
$$\tau_7 = (123), \tau_8 = (132), \alpha_1 = (12)(3), \alpha_2 = (13),$$
$$\alpha_3 = (14), \alpha_4 = (23), \alpha_5 = (24), \alpha_6 = (34).$$

The multiplication table of S_4 is shown in the appendix. One of all of subgroups of S_4 is alternating group $A_4 = \{i, \sigma_2, \sigma_5, \sigma_8, \tau_1, \tau_2, \tau_3, \tau_4, \tau_5, \tau_6, \tau_7, \tau_8\}$. There are eight nontrivial subgroups of A_4, namely $H_1 = \{i, \sigma_2, \sigma_5, \sigma_8\}, H_2 = \{i, \sigma_2\}, H_3 = \{i, \sigma_5\}, H_4 = \{i, \sigma_8\}, H_5 = \{i, \tau_1, \tau_2\}, H_6 = \{i, \tau_3, \tau_4\}$, $H_7 = \{i, \tau_5, \tau_6\}$ and $H_8 = \{i, \tau_7, \tau_8\}$ (Gardiner 1980). The diagram of poset subgroups of A_4, is as in figure 2. Next, we count the number of fuzzy subgroups of A_4. We observe that the maximal chain on that poset consists of four subgroups of A_4. Therefore, the fuzzy subgroup μ of A_4 has length 1, 2, 3 or 4. Let μ be a fuzzy subgroup of A_4. We will identify μ according to $P_1(\mu)$. The number of fuzzy subgroup of A_4 with $P_1(\mu) = H$ denoted by $o(P_1 = H)$. Every subgroup of A_4 can be chosen to be $P_1(\mu)$.

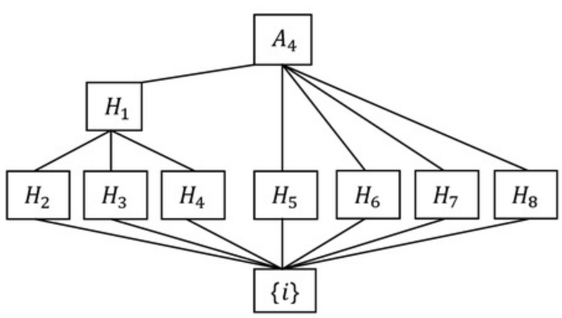

Figure 2: Diagram of poset subgroups of A_4

If $P_1(\mu) = A_4$, we only have one fuzzy subgroup of A_4, that is $\mu(x) = \theta_1, \forall x \in S_4$. If $P_1(\mu) = H_1$, then the only option we have is $A_4 = P_2(\mu)$. Therefore we only have one fuzzy subgroup of A_4 with $P_1(\mu) = H_1$. Similarly, we have one fuzzy subgroup for $P_1(\mu) = H_5$,

$P_1(\mu) = H_6$, $P_1(\mu) = H_7, P_1(\mu) = H_8$. Thus, $o(P_1 = A_4) = o(P_1 = H_5) = o(P_1 = H_6) = o(P_1 = H_7) = o(P_1 = H_8) = 1$.

If $P_1(\mu) = H_2$, we have one fuzzy subgroup of order 2 and one fuzzy subgroup of order 3, namely $\mu(x) = \begin{cases} \theta_1 & x \in H_2 \\ \theta_2 & x \in A_4 \setminus H_2 \end{cases}$ and $\mu(x) = \begin{cases} \theta_1 & x \in H_2 \\ \theta_2 & x \in H_1 \setminus H_2 \\ \theta_3 & x \in A_4 \setminus H_2 \end{cases}$.

Similarly, we have two fuzzy subgroups for $P_1(\mu) = H_2$, $P_1(\mu) = H_3$. Thus, $o(P_1 = H_2) = o(P_1 = H_3) = o(P_1 = H_4) = 2$.

Following the procedure above, we can check that $o(P_1 = \{i\}) = 12$.

Thus, the total number of fuzzy subgroups of A_4 is $(6 \times 1) + (3 \times 2) + 12 = 24$.

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Appendix: Multiplication Table of S_4

	i	σ_1	σ_2	σ_3	σ_4	σ_5	σ_6	σ_7	σ_8	σ_9	τ_1	τ_2	τ_3	τ_4	τ_5	τ_6	τ_7	τ_8	α_1	α_2	α_3	α_4	α_5	α_6
i	i	σ_1	σ_2	σ_3	σ_4	σ_5	σ_6	σ_7	σ_8	σ_9	τ_1	τ_2	τ_3	τ_4	τ_5	τ_6	τ_7	τ_8	α_1	α_2	α_3	α_4	α_5	α_6
σ_1	σ_1	σ_2	σ_3	i	τ_6	α_2	τ_2	τ_4	α_5	τ_8	σ_7	α_3	σ_4	α_1	σ_9	α_4	σ_6	α_6	τ_1	σ_8	τ_7	τ_3	σ_5	τ_5
σ_2	σ_2	σ_3	i	σ_1	α_4	σ_8	α_3	α_1	σ_5	α_6	τ_4	τ_7	τ_6	τ_1	τ_8	τ_3	τ_2	τ_5	σ_7	α_5	σ_6	σ_4	α_2	σ_9
σ_3	σ_3	i	σ_1	σ_2	τ_3	α_5	τ_7	τ_1	α_2	τ_5	α_1	σ_6	α_4	σ_7	α_6	σ_4	α_3	σ_9	τ_4	σ_5	τ_2	τ_6	σ_8	τ_8
σ_4	σ_4	τ_8	α_3	τ_1	σ_5	σ_6	i	τ_6	α_4	τ_3	α_2	σ_9	α_1	σ_1	σ_3	α_6	σ_7	α_5	τ_2	τ_5	σ_8	σ_2	τ_4	τ_7
σ_5	σ_5	α_5	σ_8	α_2	σ_6	i	σ_4	α_6	σ_2	α_1	τ_5	τ_3	τ_2	τ_8	τ_1	τ_7	τ_6	τ_4	σ_9	σ_3	α_4	α_3	σ_1	σ_7
σ_6	σ_6	τ_4	α_4	τ_5	i	σ_4	σ_5	τ_7	α_3	τ_2	σ_3	α_1	σ_9	α_5	α_2	σ_7	α_6	σ_1	τ_3	τ_1	σ_2	σ_8	τ_8	τ_6
σ_7	σ_7	τ_6	α_6	τ_7	τ_1	α_1	τ_4	σ_8	σ_9	i	α_3	σ_1	σ_3	α_4	σ_6	α_2	α_5	σ_4	σ_2	τ_2	τ_8	τ_5	τ_3	σ_5
σ_8	σ_8	α_2	σ_5	α_5	α_3	σ_2	α_4	σ_9	i	σ_7	τ_8	τ_6	τ_7	τ_5	τ_4	τ_2	τ_3	τ_1	α_6	σ_1	σ_4	σ_6	σ_3	α_1
σ_9	σ_9	τ_2	α_1	τ_3	τ_8	α_6	τ_5	i	σ_7	σ_8	σ_4	α_2	α_5	σ_6	α_4	σ_1	σ_3	α_3	σ_5	τ_6	τ_1	τ_4	τ_7	σ_2
τ_1	τ_1	σ_4	τ_8	α_3	α_1	τ_4	σ_7	α_2	τ_5	σ_3	τ_2	i	σ_2	τ_6	τ_7	σ_5	σ_8	τ_3	σ_1	σ_6	σ_9	α_6	α_4	α_5
τ_2	τ_2	α_1	τ_3	σ_9	σ_1	τ_6	α_2	σ_6	τ_7	α_3	i	τ_1	τ_8	σ_5	σ_8	τ_4	τ_5	σ_2	σ_4	σ_7	σ_3	α_5	α_6	α_4
τ_3	τ_3	σ_9	τ_2	α_1	α_5	τ_7	σ_3	σ_4	τ_6	α_4	σ_5	τ_5	τ_4	i	σ_2	τ_8	τ_1	σ_8	σ_6	α_6	α_2	σ_1	σ_7	α_3
τ_4	τ_4	α_4	τ_5	σ_6	σ_7	τ_1	α_1	α_5	τ_8	σ_1	τ_7	σ_2	i	τ_3	τ_2	σ_8	σ_5	τ_6	σ_3	α_3	α_6	σ_9	σ_4	α_2
τ_5	τ_5	σ_6	τ_4	α_4	σ_9	τ_8	α_6	σ_3	τ_1	α_2	τ_3	σ_5	σ_8	τ_7	τ_6	i	σ_2	τ_2	α_5	σ_4	α_1	σ_7	α_3	σ_1
τ_6	τ_6	α_6	τ_7	σ_7	α_2	τ_2	σ_1	α_4	τ_3	σ_4	σ_8	τ_8	τ_1	σ_2	i	τ_5	τ_4	σ_5	α_3	σ_9	α_5	σ_3	α_1	σ_6
τ_7	τ_7	σ_7	τ_6	α_6	σ_3	τ_3	α_5	α_3	τ_2	σ_6	σ_2	τ_4	τ_5	σ_8	σ_5	τ_1	τ_8	i	α_4	α_1	σ_1	α_2	σ_9	σ_4
τ_8	τ_8	α_3	τ_1	σ_4	α_6	τ_5	σ_9	σ_1	τ_4	α_5	τ_6	σ_8	σ_5	τ_2	τ_3	σ_2	i	τ_7	α_2	α_4	σ_7	α_1	σ_6	σ_3
α_1	α_1	τ_3	σ_9	τ_2	τ_4	σ_7	τ_1	σ_5	α_6	σ_2	σ_6	σ_3	σ_1	σ_4	α_3	α_5	α_2	α_4	i	τ_7	τ_5	τ_8	τ_6	σ_8
α_2	α_2	σ_5	α_5	σ_8	τ_2	σ_1	τ_6	τ_5	σ_3	τ_1	σ_9	σ_4	α_3	α_6	σ_7	σ_6	α_4	α_1	τ_8	i	τ_3	τ_7	σ_2	τ_4
α_3	α_3	τ_1	σ_4	τ_8	σ_2	α_4	σ_8	τ_2	σ_6	τ_7	σ_1	σ_7	α_6	α_2	α_5	α_1	σ_9	σ_3	τ_6	τ_4	i	σ_5	τ_5	τ_3
α_4	α_4	τ_5	σ_6	τ_4	σ_8	α_3	σ_2	τ_3	σ_4	τ_6	α_5	α_6	σ_7	σ_3	σ_1	σ_9	α_1	α_2	τ_7	τ_8	σ_5	i	τ_1	τ_2
α_5	α_5	σ_8	α_2	σ_5	τ_7	σ_3	τ_3	τ_8	σ_1	τ_4	α_6	α_4	σ_6	σ_9	α_1	α_3	σ_4	σ_7	τ_5	σ_2	τ_6	τ_2	i	τ_1
α_6	α_6	τ_7	σ_7	τ_6	τ_5	σ_9	τ_8	σ_2	α_1	σ_5	α_4	α_5	α_2	α_3	σ_4	σ_3	σ_1	σ_6	σ_8	τ_3	τ_4	τ_1	τ_2	i

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